UO‘K 539.3

UCH FAZALI INERSIONAL-VISKOELASTIK MUHIT TEBRANISH TENGLAMASI UCHUN CHEGARAVIY MASALA

Mirzoev Akmal Ahadovich,
O‘zbekiston Fanlar akademiyasi
M.T. O‘razboyev nomidagi Mexanika va
inshootlar seysmik mustahkamligi instituti
professori, f.-m.f.d.
ORCID: 0000-0002-4620-0134
Ahmadov Ilhom Aktam o‘g‘li,
Navoiy davlat universiteti tayanch doktoranti
ORCID: 0009-0008-8037-1966,
ilhomahmadov0516@gmail.com
Isomova Sabohat Islom qizi,
Navoiy davlat universiteti tayanch doktoranti
ORCID: 0009-0002-2093-3142,
isomovasabohat150@gmail.com

Annotatsiya. *Ushbu maqolada uch fazali inersional-viskoelastik muhitda tebranish jarayonlarini ifodalovchi to‘rtinchi tartibli gibrid xususiy hosilali differensial tenglama uchun birinchi tur chegaraviy va boshlang‘ich masala tadqiq qilingan. Retardatsiya vaqtlari hamda inersiya parametrlarini hisobga oluvchi reologik model uchun chegaralangan sohada matematika-fizika masalasi qo‘yilgan. Masalaning analitik yechimi o‘zgaruvchilarni ajratish (Furye) usuli yordamida qurilgan. Vaqtga bog‘liq tenglamaning xarakteristik ildizlari va so‘nish rejimlarining reologik koeffitsiyentlarga bog‘liqligi chuqur tahlil qilingan. Yechim cheksiz trigonometrik Furye qatori ko‘rinishida shakllantirilib, uning tekis yaqinlashishi hamda energik integral usuli yordamida yechimning yagonaligi qat’iy isbotlangan. Olingan analitik natijalar kon-metallurgiya sanoatidagi pulpa va shlam quvurlari hamda gidrotizimlarda nostatsionar tebranish jarayonlarini hisoblashda amaliy ahamiyatga ega.*

Kalit so‘zlar: *inersional-viskoelastik muhit, reologik model, retardatsiya vaqti, tebranish tenglamasi, Furye usuli, energik integral, yagonalik teoremasi.*

КРАЕВАЯ ЗАДАЧА ДЛЯ УРАВНЕНИЯ КОЛЕБАНИЙ ТРЁХФАЗНОЙ ИНЕРЦИОННО-ВЯЗКОУПРУГОЙ СРЕДЫ

Аннотация. *В данной работе исследована начально-краевая задача первого рода для гибридного дифференциального уравнения в частных производных четвёртого порядка, описывающего колебательные процессы в трёхфазной инерционно-вязкоупругой среде. Для реологической модели, учитывающей времена запаздывания и параметры инерции, поставлена задача математической физики в ограниченной области. Аналитическое решение задачи построено методом разделения переменных (методом Фурье). Подробно проанализирована зависимость характеристических корней временного уравнения и режимов затухания от реологических коэффициентов. Решение сформулировано в виде бесконечного тригонометрического ряда Фурье, а его равномерная сходимость и единственность строго доказаны с использованием метода энергетического интеграла. Полученные аналитические результаты имеют практическое значение для расчёта нестационарных колебательных процессов в пульпопроводах, шламовых потоках и гидравлических системах горно-металлургической отрасли.*

Ключевые слова: *инерционно-вязкоупругая среда, реологическая модель, время запаздывания, уравнение колебаний, метод Фурье, энергетический интеграл, теорема единственности.*

BOUNDARY VALUE PROBLEM FOR THE OSCILLATION EQUATION OF A THREE-PHASE INERTIAL-VISCOELASTIC MEDIUM

Abstract. *This paper investigates a first-type initial-boundary value problem for a fourth-order hybrid partial differential equation describing oscillation processes in a three-phase inertial-viscoelastic medium. A mathematical physics problem is posed in a bounded domain for a rheological model incorporating*

retardation times and inertia parameters. The analytical solution of the problem is constructed using the method of separation of variables (Fourier method). The dependence of the characteristic roots of the time-dependent equation and damping regimes on rheological coefficients is thoroughly analyzed. The solution is formulated as an infinite trigonometric Fourier series, and its uniform convergence together with uniqueness is rigorously proved via the energy integral method. The obtained analytical results are of practical importance for calculating non-stationary oscillation processes in pulp and slurry pipelines as well as hydraulic systems in the mining and metallurgical industry.

Keywords: *inertial-viscoelastic medium, rheological model, retardation time, oscillation equation, Fourier method, energy integral, uniqueness theorem.*

Kirish. Tutash muhitlar mexanikasida yopishqoq-elastik (viskoelastik) xossaga ega bo‘lgan materiallarning dinamik holatini hamda ulardagi tebranish jarayonlarini modellashtirish amaliy va nazariy jihatdan muhim masalalardan biridir. Xususan, kon-metallurgiya sanoatidagi quyuq pulpa va shlam quvurlarida, neft-gaz muhandisligidagi burg‘ilash eritmalarida, seysmik to‘lqinlar ta’siridagi nam g‘ovak grunt qatlamlarida va polymer tizimlarda to‘lqinlar tarqalishini aniqroq tasvirlash uchun muhitning nafaqat elastik va yopishqoq xossalarini, balki uning inersional xarakteristikalarini ham hisobga olish talab etiladi [1, 2].

Muhitning relaksatsiya va retardatsiya jarayonlari klassik reologik modellar - Kelvin-Voigt va Maksvell modellarida keng o‘rganilgan bo‘lsa-da, yuqori chastotali hamda jadallashtirilgan tebranish rejimlarida massaning inersiyaga ko‘rsatadigan qarshiligi to‘liq qamrab olinmaydi. Bu yo‘nalishda R.M. Christensen [1], A. Wineman [2] va F. Mainardi [3] chiziqli viskoelastik muhitlarda to‘lqinlar dinamikasi nazariyasini rivojlantirganlar. A.A. Il'yushin va B.E. Pobedrya [4] thermoviskoelastik muhitlar holat tenglamalarining matematik asoslarini yaratgan bo‘lsa, O.A. Ladyzhenskaya [5] hamda K. Khudoynazarov va A. Abdurazaqov [6] viskoelastik konstruksiya elementlarida differensial tenglamalar uchun chegaraviy masalalarning to‘g‘ri qo‘yilganligini (korrektligini) isbotlab berishgan.

Biroq yuqorida keltirilgan ishlarda massa inersiyasining ta'siri asosan tenglamaning chap tomonidagi tashqi kuchlar muvozanatida ($\rho\frac{\partial^2 u}{\partial t^2}$) hisobga olingan bo‘lib, reologik modelning ichki tuzilmasida (so‘nish va retardatsiya mexanizmida) inersion elementlarning tutgan o‘rni yetarlicha ochib berilmagan.[8-10]

Ushbu maqolaning maqsadi - uch fazali inersional-viskoelastik muhit tebranishi uchun gibrid 4-tartibli xususiy hosilali differensial tenglamaga keltiriladigan birinchi tur chegaraviy va boshlang‘ich masalani qo‘yish hamda uni o‘zgaruvchilarni ajratish (Furye) usulida analitik yechishdan iborat. Shuningdek, Furye qatorining tekis yaqinlashishi va energik integral usuliga tayanib, yechimning mavjudligi hamda yagonaligini isbotlash, reologik parametrlarning tebranish so‘nish koeffitsiyentiga ta’sirini tahlil qilish nazarda tutilgan.

Metodlar. Uch fazali (elastik, viskoz va inersional) reologik muhitning deformatsiyalanish jarayonini modellashtirishda elementlarning parallel ulangan sxemasi qaraladi. Muhitdagi umumiy kuchlanish $\sigma(t)$ elastiklik (f_3G_i), viskozlik ($f_1\mu_i$) hamda inersional-gidrodinamik (f_2m_{li}) tashkil etuvchilarning yig‘indisidan iborat.[11,12]

$$\sigma(t) = \sigma_e + \sigma_q + \sigma_i \tag{1}$$

Deformatsiya $\varepsilon(t)$ va uning vaqt bo‘yicha hosilalari orqali ushbu bog‘lanish quyidagi differensial ko‘rinishni oladi:[o‘zimniki]

$$\sigma(t) = f_3G_i\varepsilon(t) + f_1\mu_i\frac{\partial\varepsilon(t)}{\partial t} + f_2m_{li}\frac{\partial^2\varepsilon(t)}{\partial t^2} \tag{2}$$

Tenglamadan elastiklik koeffitsiyenti f_3G_i ni qavsdan tashqariga chiqarib, Kelvin-Voigt bo‘yicha retardatsiya vaqti $t_{ret}^K = \frac{f_1\mu_i}{f_3G_i}$ hamda inersional-viskoelastik retardatsiya vaqti $\left(t_{ret}^{Kh2}\right)^2 = \frac{f_2m_{li}}{f_3G_i}$ belgilashlarini kiritsak, muhitning holat tenglamasi hosil bo‘ladi.

$$\sigma(t) = f_3G_i\left(\varepsilon(t) + t_{ret}^K\frac{\partial\varepsilon(t)}{\partial t} + \left(t_{ret}^{Kh2}\right)^2\frac{\partial^2\varepsilon(t)}{\partial t^2}\right) \tag{3}$$

Muhitning deformatsiya orqali ifodalangan dinamik tenglamasini hosil qilish uchun tutash muhitlar harakat tenglamasi $\rho\frac{\partial^2 u}{\partial t^2} = \frac{\partial\sigma}{\partial x}$ va kichik deformatsiyalar nisbati $\varepsilon = \frac{\partial u}{\partial x}$ dan foydalanamiz. Natijada x va t o‘zgaruvchilariga nisbatan 4-tartibli aralash xususiy hosilali differensial tenglama kelib chiqadi.

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial x^2} + t_{ret}^K\frac{\partial^3 u}{\partial x^2\partial t} + \left(t_{ret}^{Kh2}\right)^2\frac{\partial^4 u}{\partial x^2\partial t^2} \tag{4}$$

bu yerda $a^2 = \frac{f_3 G_i}{\rho} > 0$ - muhit elastik to‘lqinlarining tarqalish tezligi kvadratini, $t_{ret}^K > 0$ va $t_{ret}^{Kh2} > 0$ esa mos ravishda viskoz va inersional retardatsiya vaqtlarini ifodalaydi.

Chegaralangan $D_T = \{(x,t): 0 < x < l,\ 0 < t < T\}$ sohada (4) tebranish tenglamasi uchun birinchi tur (Dirixle) chegaraviy shartlari qo‘yiladi.

$$u(0,t) = 0, \quad u(l,t) = 0, \quad t \geq 0 \tag{5}$$

Bundan tashqari $t = 0$ vaqt momentidagi boshlang‘ich shart qo‘shiladi.

$$u(x,0) = \phi(x), \quad \left.\frac{\partial u(x,t)}{\partial t}\right|_{t=0} = \psi(x), \quad 0 \leq x \leq l \tag{6}$$

Boshlang‘ich va chegaraviy shartlar birga qo‘llanganda uzluksizlik saqlanishi uchun ushbu moslashuv shartlari bajarilishi kerak.

$$\phi(0) = \phi(l) = 0, \quad \phi''(0) = \phi''(l) = 0, \quad \psi(0) = \psi(l) = 0 \tag{7}$$

(4) tenglamani (6) masalasining klassik yechimi mavjudligi, yagonaligi hamda qurilgan Furye qatorining tekis yaqinlashishi tekshirib ko‘rilishi zarur. Buning quyidagi teorema va lemmalardan foydalanamiz.

Teorema 1. *Agar* $\phi(x) \in C^3[0,l]$ *va* $\psi(x) \in C^2[0,l]$ *funksiyalar (7) moslashuv shartlarini qanoatlantirsa, u holda (4)-(6) masalaning* $u(x,t) \in C^{2,2}(Q_T)$ *klassik yechimi mavjud, yagona va u ushbu Furye qatori ko‘rinishida ifodalanadi:*

$$u(x,t) = \sum_{n=1}^{\infty} T_n(t) \sin\frac{n\pi x}{l} \tag{8}$$

Isbot. Yechimni $u(x,t) = X(x)T(t)$ ko‘rinishida izlasak (4) tenglama quyidagi ko‘rinishga o‘tadi.

$$\frac{T''(t)}{a^2 T(t) + t_{ret}^K T'(t) + \left(t_{ret}^{Kh2}\right)^2 T''(t)} = \frac{X''(x)}{X(x)} = -\lambda \tag{9}$$

Natijada ikkita ajralgan differensial tenglama va (5) chegaraviy shartlarga ko‘ra Shturm-Liuvill masalasi hosil bo‘ladi: $X''(x) + \lambda X(x) = 0, X(0) = 0, X(l) = 0$

Ushbu masalaning xos qiymatlari va ortogonal xos funksiyalari:

$$\lambda_n = \left(\frac{n\pi}{l}\right)^2, \quad X_n(x) = \sin\frac{n\pi x}{l}, \quad n = 1,2,3,\ldots \tag{10}$$

Vaqtga bog‘liq $T_n(t)$ funksiya uchun 2-tartibli oddiy differensial tenglama keltirib chiqariladi.

$$\left(1 + a^2\lambda_n\left(t_{ret}^{Kh2}\right)^2\right) T_n''(t) + a^2\lambda_n t_{ret}^K T_n'(t) + a^2\lambda_n T_n(t) = 0 \tag{11}$$

(6) boshlang‘ich shartlardan $T_n(t)$ uchun boshlang‘ich shartlarni topamiz.

$$T_n(0) = \phi_n = \frac{2}{l}\int_0^l \phi(x) \sin\frac{n\pi x}{l}\,dx, \quad T_n'(0) = \psi_n = \frac{2}{l}\int_0^l \psi(x) \sin\frac{n\pi x}{l}\,dx \tag{12}$$

(11) tenglamaning xarakteristik tenglamasini tuzsak (13) hosil bo‘ladi.

$$\left(1 + a^2\lambda_n\left(t_{ret}^{Kh2}\right)^2\right) r^2 + a^2\lambda_n t_{ret}^K r + a^2\lambda_n = 0 \tag{13}$$

Barcha reologik parametrlar $a^2, t_{ret}^K, t_{ret}^{Kh2} > 0$ va $\lambda_n > 0$ bo‘lgani uchun, xarakteristik ildizlar $r_{1,2}^{(n)} = -\gamma_n \pm i\omega_n$ manfiy haqiqiy qismga ega ($\gamma_n > 0$), bu esa tebranishlarning vaqt o‘tishi bilan so‘nishini ko‘rsatadi. (11)-(12) masalaning analitik yechimi (14) ko‘rinishda bo‘ladi.

$$T_n(t) = e^{-\gamma_n t}\left(\phi_n \cos\omega_n t + \frac{\psi_n + \gamma_n\phi_n}{\omega_n}\sin\omega_n t\right) \tag{14}$$

Teorema 1. shartiga ko‘ra $\phi(x) \in C^3[0,l]$ va $\psi(x) \in C^2[0,l]$ bo‘lgani uchun, bo‘laklab integrallash usulini qo‘llab, Furye koeffitsiyentlarini topa olamiz.

$$\phi_n = \frac{1}{\lambda_n^{3/2}}\phi_n^{(3)}, \quad \psi_n = \frac{1}{\lambda_n}\psi_n^{(2)} \tag{15}$$

bu yerda $\sum_{n=1}^{\infty}\left(\phi_n^{(3)}\right)^2 < \infty$ va $\sum_{n=1}^{\infty}\left(\psi_n^{(2)}\right)^2 < \infty$. Veyershtrass va Koshi-Bunyakovskiy tengsizliklariga ko‘ra:

$$|u(x,t)| \leq \sum_{n=1}^{\infty} |T_n(t)| \leq M\sum_{n=1}^{\infty}\frac{1}{\lambda_n^{3/2}}\left(|\phi_n^{(3)}| + |\psi_n^{(2)}|\right) \leq M_1\sum_{n=1}^{\infty}\frac{1}{n^3} < \infty \tag{16}$$

Xuddi shunday, $\frac{\partial^2 u}{\partial x^2}$, $\frac{\partial^2 u}{\partial t^2}$ va $\frac{\partial^4 u}{\partial x^2 \partial t^2}$ hosilalardan tuzilgan qatorlar tekis yaqinlashishi va $u(x,t) \in C^{2,2}(Q_T)$ ekanligi isbotlanadi.

Endi yechimni yagonaligi haqidagi qismi ko‘rib chiqamiz. Faraz qilaylik, masalaning ikkita $u_1(x,t)$ va $u_2(x,t)$ klassik yechimlari mavjud bo‘lsin. Ularning farqini $w(x,t)=u_1(x,t)-u_2(x,t)$ deb belgilaymiz. $w(x,t)$ funksiya bir jinsli (4) tenglamani hamda nolga teng chegaraviy va boshlang‘ich shartlarni qanoatlantiradi.

Masala uchun quyidagi integralni qaraymiz:

$$E(t)=\frac{1}{2}\int_0^l\left[\left(\frac{\partial w}{\partial t}\right)^2+a^2\left(\frac{\partial w}{\partial x}\right)^2+a^2\left(t_{ret}^{Kh2}\right)^2\left(\frac{\partial^2 w}{\partial x\partial t}\right)^2\right]dx \tag{17}$$

Ushbu $E(t)$ integralining vaqt bo‘yicha hosilasini olib, (4) tenglamani o‘rniga qo‘yamiz va bo‘laklab integrallaymiz:

$$\frac{dE(t)}{dt}=-a^2t_{ret}^{K}\int_0^l\left(\frac{\partial^2 w}{\partial x\partial t}\right)^2dx\le 0 \tag{18}$$

Bu $E(t)$ funksiyaning o‘smaydigan ekanligini ko‘rsatadi ($0\le E(t)\le E(0)$). Nol boshlang‘ich shartlarga ko‘ra $E(0)=0$, demak barcha $t\ge 0$ uchun $E(t)\equiv 0$. Bunga ko‘ra $\int_0^l\left(\frac{\partial w}{\partial x}\right)^2dx=0$ va $w(0,t)=0$ shartdan $w(x,t)\equiv 0$, ya’ni $u_1(x,t)\equiv u_2(x,t)$ kelib chiqadi. Teorema to‘liq isbotlandi.

Natija va muhokama.

1-Teoremaga muvofiq, uch fazali inersional-viskoelastik muhit tebranishini ifodalovchi (4)-(6) chegaraviy masalaning analitik yechimi quyidagi cheksiz trigonometric Furye qatori (8) ko‘rinishida shakllantiriladi. Bu yerda vaqtinchalik $T_n(t)$ funksiya (11) tenglamaning analitik yechimi bo‘lib, u muhitning reologik va inersional xarakteristikalariga bevosita bog‘liq bo‘ladi. (11) tenglamaning D_n diskriminantining ishorasi tebranishlarning fizik xarakterini belgilab beradi:

$$D_n=a^4\lambda_n^2(t_{ret}^{K})^2-4a^2\lambda_n\left(1+a^2\lambda_n\left(t_{ret}^{Kh2}\right)^2\right) \tag{19}$$

Parametrlarning nisbatiga qarab uch xil fizik-mexanik rejim bo‘lishi mumkin: ($D_n<0$) Ko‘pgina amaliy diskriminant manfiy bo‘ladi. Xarakteristik ildizlar kompleks qo‘shma ko‘rinishga ega ($r_{1,2}^{(n)}=-\gamma_n\pm i\omega_n$):

$$\gamma_n=\frac{a^2\lambda_n t_{ret}^{K}}{2\left(1+a^2\lambda_n\left(t_{ret}^{Kh2}\right)^2\right)},$$
$$\omega_n=\frac{\sqrt{4a^2\lambda_n\left(1+a^2\lambda_n\left(t_{ret}^{Kh2}\right)^2\right)-a^4\lambda_n^2(t_{ret}^{K})^2}}{2\left(1+a^2\lambda_n\left(t_{ret}^{Kh2}\right)^2\right)} \tag{20}$$

Ushbu holdagi yechim (22) ko‘rinishda bo‘lib o‘zgarmas koeffitsentli, bir jinsli differensial tenglamani yechimi ko‘rinishda ifodalanadi.

$$T_n(t)=e^{-\gamma_n t}\left(\phi_n\cos\omega_n t+\frac{\psi_n+\gamma_n\phi_n}{\omega_n}\sin\omega_n t\right) \tag{21}$$

($D_n>0$) Nisbatan yuqori yopishqoqlik va kichik inersiyada xarakteristik ildizlar haqiqiy va manfiy bo‘ladi. Tizim tebranmasdan, eksponentasimon ravishda muvozanat holatiga qaytadi. ($D_n=0$) Ikkala ildiz ham bir-biriga teng va manfiy bo‘lganda, kretik holat yuz beradi.

Klassik Kelvin-Voigt modelida ($t_{ret}^{Kh2}=0$) so‘nish koeffitsiyenti $\gamma_n^{(KV)}=\frac{1}{2}a^2\lambda_n t_{ret}^{K}$ bo‘lib, garmoniklik tartibi n ortishi bilan so‘nish tezligi kvadratik ravishda o‘sib ketadi. Uch fazali inersional-viskoelastik modelda esa inersion parametr $\left(t_{ret}^{Kh2}\right)^2$ mahrajda qatnashgani sababli, yuqori chastotali tebranishlarda asimptotik cheklanishlar yuz beradi.

$$\lim_{n\to\infty}\gamma_n=\lim_{n\to\infty}\frac{a^2\lambda_n t_{ret}^{K}}{2\left(1+a^2\lambda_n\left(t_{ret}^{Kh2}\right)^2\right)}=\frac{t_{ret}^{K}}{2\left(t_{ret}^{Kh2}\right)^2}=\text{const} \tag{22}$$

Xulosa. Ushbu maqolada uch fazali (elastik, viskoz va inersional) reologik model asosida quriladigan, tebranish jarayonlarini ifodalovchi to‘rtinchi tartibli gibrid xususiy hosilali differensial tenglama uchun birinchi tur chegaraviy va boshlang‘ich masala qo‘yildi hamda o‘zgaruvchilarni ajratish usuli yordamida to‘liq analitik tadqiq qilindi. Olib borilgan tadqiqot natijasida quyidagi asosiy xulosalarga kelindi: Muhitning holat tenglamasiga inersion tashkil etuvchi - f_2m_{li} elementining kiritilishi klassik Kelvin-Voigt modelini umumlashtirib, muhitning reologik xatti-harakatini ikkita retardatsiya vaqti - viskoz t_{ret}^{K} va inersional-viskoelastik t_{ret}^{Kh2} orqali tavsiflashga imkon berdi. Vaqtga bog‘liq (11) oddiy differensial tenglamaning

xarakteristik ildizlari tahlili natijasida tebranish jarayonining uchta fizik-mexanik rejimi ($D_n < 0$ so‘nuvchi tebranma rejim, $D_n > 0$ aperiodik so‘nish, $D_n = 0$ kritik holat) reologik parametrlarning o‘zaro nisbatiga bog‘liq holda aniq belgilandi. Masalaning (8) ko‘rinishidagi Furye qatori shaklidagi yechimi qurilib, Veyershtrass va Koshi-Bunyakovskiy tengsizliklari asosida uning tekis yaqinlashishi, shuningdek integral usuli ((17)-(18)) yordamida yechimning yagonaligi isbotlandi va Teorema 1. orqali klassik yechimning mavjudligini ham kafolatladi. Eng muhim natija sifatida, inersion parametr $\left(t_{ret}^{Kh2}\right)^2$ning so‘nish koeffitsiyenti γ_n ifodasining maxrajida qatnashishi aniqlandi (22) bu klassik Kelvin-Voigt modelidan farqli o‘laroq, garmonikalar tartibi $n \to \infty$ da so‘nish koeffitsiyentining cheksiz o‘sishining oldini olib, uni chekli qiymatga $(t_{ret}^{K}/2\left(t_{ret}^{Kh2}\right)^2$ intiltiradi. Demak, inersion elementning modelga kiritilishi yuqori chastotali tebranishlarda jarayonning fizik jihatdan reallikka yaqinroq tasvirlanishini ta'minlaydi.

Olingan analitik natijalar kon-metallurgiya sanoatidagi quyuq pulpa va shlam quvurlari, gidrotizimlar, neft-gaz muhandisligidagi burg‘ilash eritmalari hamda seysmik ta‘sir ostidagi nam g‘ovak grunt qatlamlaridagi nostatsionar tebranish jarayonlarini sonli modellashtirish va muhandislik hisob-kitoblarida amaliy qo‘llanilishi mumkin. Kelgusi tadqiqotlarda ushbu model ikki va uch o‘lchamli sohalarga, shuningdek nochiziqli reologik bog‘lanishlar va o‘zgaruvchan kesim yuzali quvurlar holatiga umumlashtirilishi maqsadga muvofiqdir.

ADABIYOTLAR:

1. Christensen R.M. Theory of Viscoelasticity: An Introduction. – New York: Academic Press, 1982.

2. Wineman A. Some perspectives on mathematical modeling of viscoelastic behavior. // International Journal of Non-Linear Mechanics, vol. 44, pp. 837-846, (2009).

3. Mainardi F. Fractional Calculus and Waves in Linear Viscoelasticity: An Introduction to Mathematical Models. – London: Imperial College Press, 2010.

4. Ильюшин А.А., Победря Б.Е. Основы математической теории термовязкоупругости. – М.: Наука, 1970.

5. Ladyzhenskaya O.A. The Boundary Value Problems of Mathematical Physics. – New York: Springer-Verlag, 1985.

6. Khudoynazarov K., Abdurazaqov A. Non-stationary waves in a viscoelastic plate on an elastic foundation. // Journal of Applied Mathematics and Mechanics, vol. 82, pp. 210-218, (2018).

7. Navruzov K., Mirzoev A., Ahmadov I. Comparison of the exact and asymptotic solutions to the problem of oscillatory flow of a viscous fluid in a cylindrical pipe // Raqamli Transformatsiya va Sun’iy Intellekt ilmiy jurnali. 2025. Vol. 3, Issue 5. - P.272-277. ISSN 3030-3346.

8. Navruzov K., Rajabov S., Abdikarimov N., Turdimuradov O., Nasrulloeva R. Non-stationary flow of viscoelastic fluid in a cylindrical pipe // E3S Web of Conferences. 2024. Vol. 587. Article 01013. P. 1-11. DOI: 10.1051/e3sconf/202458701013.

9. Navruzov K., Sharipova Sh. B. Oscillatory flow of rheological complex fluid in a flat channel // E3S Web of Conferences. 2024. Vol. 508. 06008. P. 1-9. DOI: 10.1051/e3sconf/202450806008.

10. Ding Z., Jian Y. Electrokinetic oscillatory flow and energy conversion of viscoelastic fluids in microchannels: a linear analysis // J. Fluid Mech. 2021. Vol. 919. A20. DOI: 10.1017/jfm.2021.380.

11. Casanellas L., Ortín J. Laminar oscillatory flow of Maxwell and Oldroyd-B fluids: Theoretical analysis // Journal of Non-Newtonian Fluid Mechanics. 2011. Vol. 166, No. 23-24. P. 1315-1326. DOI: 10.1016/j.jnnfm.2011.08.010.

12. Ahmadov I. A. Yopishqoqlikni oshiruvchi reagent ta’sirida sanoat suspenziyasi oquvchanlik xususiyatlarining eksperimental va sonli tahlili // [jurnal nomi]. – 2026. – № 2. – DOI: 10.36522/2181-4317/9-2-2026-7. – EDN: RQQKJH.