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# NOCHIZIQLI CHEGARAVIY SHARTLAR BILAN BOG‘LANGAN KO‘P O‘LCHOVLI CHIZIQSIZ ISSIQLIK TARQALISH MASALASI YECHIMLARINING XOSSALARI

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***Annotatsiya.*** *Ushbu maqolada nochiziqli chegaraviy shartlar bilan bog‘langan ko‘p o‘lchovli chiziqsiz issiqlik tarqalish masalasi yechimlarining sifat xossalari tadqiq etilgan. Tez diffuziya holida solishtirish prinsipidan foydalanilgan holda, sistema yechimlarining vaqt bo‘yicha global bo‘lish yoki chegaralanmaganlik shartlari topilgan. Olingan nazariy natijalar asosida, saqlanish qonunlariga muvofiq balans usuli yordamida ko‘p o‘lchovli issiqlik tarqalish masalasi uchun oshkormas ayirmali sxema va iteratsion jarayon qurilgan, bunda iteratsion jarayon uchun boshlang‘ich yaqinlashish sifatida avtomodel yechimlarning asimptotikalaridan foydalanish taklif etilgan. Hisoblash eksperimenti o‘tkazilib, sonli parametrlarning turli qiymatlarda olingan sonli natijalar grafiklar ko‘rinishida tasvirlangan.*

***Kalit so‘zlar:*** *global yechim, asimptotika, chegaralanmagan yechim, avtomodel tenglama, chiziqsiz chegaraviy shart, Fujita turidagi kritik ko‘rsatkichlar, mavjudlik.*

# СВОЙСТВА РЕШЕНИЙ МНОГОМЕРНОЙ ЗАДАЧИ НЕЛИНЕЙНОГО ТЕПЛОВОГО РАСПРОСТРАНЕНИЯ, СВЯЗАННОЙ С НЕЛИНЕЙНЫМИ ГРАНИЧНЫМИ УСЛОВИЯМИ

***Аннотация.*** *В статье исследуются качественные свойства решений многомерной нелинейной задачи теплопроводности с нелинейными связанными граничными условиями. Для случая быстрой диффузии с использованием принципа сравнения установлены условия глобального существования решений системы и условия их неограниченности во времени. На основе полученных теоретических результатов для многомерной задачи теплопроводности с использованием балансового метода, основанного на законах сохранения, построены неявная разностная схема и итерационный процесс. В качестве начального приближения итерационного процесса предложено использовать асимптотики автомодельных решений. Проведён вычислительный эксперимент, результаты численных расчетов при различных значениях параметров представлены в графическом виде.*

***Ключевые слова:*** *глобальное решение, асимптотика, неограниченное решение, автомодельное уравнение, нелинейное граничное условие, критические показатели типа Фуджита, существование.*

# PROPERTIES OF SOLUTIONS TO A MULTI-DIMENSIONAL NONLINEAR HEAT TRANSFER PROBLEM RELATED TO NONLINEAR BOUNDARY CONDITIONS

***Abstract.*** *This paper investigates the qualitative properties of solutions to a multidimensional nonlinear heat conduction problem with nonlinear coupled boundary conditions. For the case of fast diffusion, using the comparison principle, conditions for the global existence of solutions to the system and conditions for their unboundedness in time are established. Based on the theoretical results obtained, an implicit difference scheme and an iterative process are constructed for the multidimensional heat conduction problem using a balance method based on conservation laws. As an initial approximation for the iterative process, the asymptotics of self-similar solutions are proposed. A computational experiment is conducted, and the results of numerical calculations obtained for various parameter values are presented graphically.*

***Keywords:*** *global solution, asymptotics, unbounded solution, self-similar equation, nonlinear boundary condition, Fujita type critical parameters, existence.*

**Kirish.** Biz ushbu ishda $Q_+ = \left\{ (x,t) : x \in R_+^N, 0 < t < T \right\}$ sohada quyidagi masalani qaraymiz:

$$\begin{cases} u_{it} = \nabla\left(\left|\nabla u_i^{m_i}\right|^{p_i} \nabla u_i^{m_i}\right), & x \in R_+^N, 0 < t < T, \\ -\left|\nabla u_i^{m_i}\right|^{p_i} \dfrac{\partial u_i^{m_i}}{\partial x_1} = u_{i+1}^{q_i}, & u_{k+1} = u_1, x_1 = 0, 0 < t < T, \\ u_i(x,0) = u_{i0}(x), & x \in R_+^N, \end{cases} \tag{1}$$

bu yerda $m_i$, $p_i$ nochiziqli muhit xususiyatlarini ifodalovchi sonli parametrlar va quyidagi shartga ega: $p_i < \dfrac{1}{m_i} - 1$, $q_i > 0$, $u_{i0}(x)$ - uzluksiz, manfiy bo‘lmagan, chegaralangan funksiyalar, bunda: $-\left|\nabla u_{i0}^{m_i}\right|^{p_i} \dfrac{\partial u^{m_i}{}_{i0}}{\partial x_1} = u_{(i+1)0}^{q_i}$, $x_1 = 0$, va ba’zi $x_0 \in R_+^N$ nuqta uchun $\sup pu_{i0} \subset B_R(x_0) \cap R_+^N, u_{i0} \neq 0$.

(1) tenglamalar sistemasi buziluvchi bo‘lib, $u_i(x,t) = 0$ yoki $\left|\nabla u_i^{m_i}\right| = 0$ bo‘lganda klassik ma’noda yechimga ega bo‘lmasligi mumkin. Shuning uchun (1) masalaning $\left|\nabla u_i^{m_i}\right|^{p_i} \nabla u_i^{m_i} \in C\left(R_+^N \times (0,+\infty)\right)$, $0 < u_i^{m_i} \in C(Q)$, $(i = 1,2,...,k)$ sinfda umumlashgan yechimini qaraymiz va bu yechim (1) tenglamalar sistemasini integral ayniyat ma’nosida yoki taqsimot ma’nosida qanoatlantiradi [1, 2].

(1) masala tabiatshunoslikda uchraydigan ko‘p komponentli nochiziqli muhitlarda ko‘plab fizik, biologik jarayonlarni tavsiflaydi. Jumladan, sonli parametrlarning turli qiymatlarida o‘zaro diffuziya jarayonlarini, turli xil jismlarning o‘zaro issiqlik almashinuvini, yonish, suyuqlik va gazning politropik filtratsiyasini, biologik turlar uchun populyatsiya jarayonini kabi turli xil hodisalarni tasvirlaydi [3-4]

[6] ishda nochiziqli filtratsiya masalalarining global mavjudligi va asimptotik xatti-harakatlari ham tadqiq etilgan. Mazkur ishda bir jinsli bo‘lmagan muhitdagi quyidagi kvazichiziqli parabolik tenglama qaralgan:

$$\rho(x) u_t = \nabla\left(\left|\nabla u^m\right|^{p-2} \nabla u^m\right), (x,t) \in R_+^N \times (0,+\infty), \tag{2}$$

nochiziqli chegaraviy shart $$-\left|\nabla u^m\right|^{p-2} \frac{\partial u^m}{\partial x_1}(0,t) = u^q, \; x_1 = 0, \; t > 0, \tag{3}$$

va boshlang‘ich shart $$u(x,0) = u_0(x) \geq 0, \tag{4}$$

bilan birgalikda o‘rganilgan. Bu yerda $\rho(x) = \left(1 + |x|\right)^n -$ muhitning bir jinsli bo‘lmagan zichligini ifodalovchi funksiya. Shuningdek, tez diffuziya holati $1 < p < 1 + \dfrac{1}{m}$ uchun yechimlarning global mavjudligi, kritik global mavjudlik kritik ko‘rsatkichi va Fujita kritik ko‘rsatkichi aniqlangan hamda global yechimlar holida ularning asimptotik xatti-harakati o‘rganilgan.

[7] ishda quyidagi nochiziqli buziluvchan parabolik tipdagi tenglamalar sistemasi:

$$u_{it} = \left( u_{ix}^{m_i} |^{p_i} \left(u_i^{m_i}\right)_x\right)_x, \quad i = 1,2,\ldots,k, \quad x > 0, \quad 0 < t < T, \tag{5}$$

nochiziqli chegaraviy

$$-| u_{ix}^{m_i} |^{p_i} \left(u_i^{m_i}\right)_x (x,t) = \prod_{j=1}^{k} u_j^{q_{ij}}(0,t), \quad i = 1,2,\ldots,k, \quad 0 < t < T, \tag{6}$$

hamda boshlang‘ich shartlar bilan berilgan

$$u_i(x,0) = u_{i0}(x), \quad i = 1,2,\ldots,k, \quad x > 0, \tag{7}$$

(5)-(7) masalaning avtomodel yechimlari asimptotikasi va sonli yechimlarini tadqiq etilgan. Bu yerda $k \geq 1$, $p_i > \dfrac{1}{m_i} - 1$, $m_i \geq 1$, $q_{ij} > 0$ $(i = 1,2,...,k)$ va $u_{i0}(i = 1,2,\ldots,k)$ funksiyalar $\mathrm{R}_+$ da manfiy

bo‘lmagan uzluksiz funksiyalar. Bu ishda masalaning kompakt yurituvchili yechimi uchun $\xi_i \to \left(\frac{a_i}{b_i}\right)^{\frac{p_i+1}{p_i+2}}$ da quyidagi $F_i(\xi_i) = \overline{F}_i\left(1+o\left(1\right)\right)$ asimptotika o‘rinli bo‘lishi isbotlangan.

[8] ishda mualliflar quyidagi boshlang‘ich-chegaraviy masalani qaraganlar:

$$\begin{cases} u_t = (u^m)_{xx}, \quad (x,t) \in R_+ \times (0,T), \\ -(u^m)_x(0,t) = u^p(0,t), \quad t \in (0;T), \\ u(x,0) = u_0(x), \quad x \in R_+, \end{cases}$$

bu yerda $0 < m < 1, \quad p > 0.$ Mazkur modelda $0 < m < 1$ sharti tez diffuziyani ifodalaydi. Chegaradagi $-(u^m)_x(0,t) = u^p(0,t),$ shart esa diffuziya jarayonining nochiziqli chegaraviy manba bilan o‘zaro ta’sirini ifodalaydi. Tadqiqotning asosiy maqsadi $m$ va $p$ parametrlarining qiymatlariga qarab musbat yechimlarning vaqt bo‘yicha global mavjudligi va chekli vaqtda blow-up bo‘lish xususiyatlarini aniqlashdan iborat. Barcha yechimlar global mavjud bo‘ladigan, shuningdek global va blow-up yechimlar birgalikda mavjud bo‘lishi mumkin bo‘lgan holatlarni tavsiflaganlar.

$\rho(x) = 1, N = 1, m = 1$ bo‘lgan (2)–(4) masalaning ayrim yechim xossalari Zejja, Jingxue va Chunpeng [5] tomonidan o‘rganilgan. Sekin diffuziya holatida $0 < q \le (m+1)(p-1) - \frac{1}{p}$ bo‘lganda, (2)–(4) masalaning yechimi vaqt bo‘yicha global mavjud bo‘lishi isbotlangan. Bundan tashqari, ular $q_c = (m+1)(p-1) - 1$ kritik ko‘rsatkich ekanligini ko‘rsatganlar. Agar $(m+1)(p-1) - \frac{1}{p} < q < q_c,$ bo‘lsa, barcha yechimlar chekli vaqtda chegaralanmas bo‘lib qoladi, ammo $q > q_c$ bo‘lganda global yechimlar mavjud bo‘ladi. Tez diffuziya holatida ham shunga o‘xshash natijalar [7,8] da olingan.

**Tadqiqot metodologiyasi.**

(1) masalani yechishda radial-simmetrik almashtirish bajarib $r = \sum_{i=1}^{N} \sqrt{x_i^2}$, $\frac{\partial r}{\partial x_i} = \frac{2x_i}{2\sum_{i=1}^{N}\sqrt{x_i^2}} = \frac{x_i}{r}$ foydalanib, quyidagi radial-simmetrik ko‘rinishga keltiramiz:

$$\begin{cases} \frac{\partial u_i}{\partial t} = r^{1-N} \frac{\partial}{\partial r}\left[ r^{N-1} \left| \frac{\partial u_i^{m_i}}{\partial r} \right|^{p_i} \frac{\partial u_i^{m_i}}{\partial r} \right], \\ -\left| \frac{\partial u_i^{m_i}}{\partial r} \right|^{p_i} \frac{\partial u_i^{m_i}}{\partial r_1} = u_{i+1}^{q_i}. \end{cases} \tag{8}$$

Avtomodel tenglamalar sistemasini qurish uchun (8) masalani yechimini quyidagi ko‘rinishda qidiramiz, ya’ni:

$$\overline{u}_i(x,t) = (T+t)^{-\alpha_i} v_i(\eta_i), \qquad \eta_i = \left|\xi_i\right|, \ \xi_i = (1+x)(\tau+t)^{-\beta_i}, \tag{9}$$

bu yerda $\alpha_i + 1 = \beta_i + \left(\alpha_i m_i + \beta_i\right)(p_i+1), \ (\alpha_i m_i + \beta_i)(p_i+1) = \alpha_{i+1} q_i$ , $(i = 1,2,...,k).$

(8) masala quyidagi ko‘rinishga keladi.

$$\begin{cases} \eta^{1-N}\left(\eta^{N-1}\left|\left(v_i^{m_i}\right)'\right|^{p_i}\left(v_i^{m_i}\right)'\right)'(\eta_i)+\alpha_i v_i(\eta_i)+\eta_i\beta_i v_i'(\eta_i)=0, \\ -\left|\left(v_i^{m_i}\right)'\right|^{p_i}\dfrac{\partial v_i^{m_i}}{\partial \eta_1}=v_{i+1}^{q_i}(\eta_{i+1}), \qquad i=1,2,...,k. \end{cases} \tag{10}$$

(10) masalani avtomodel yechimi $\bar{v}_i(\eta_i)=\left(a_i+b_i\eta_i^{\gamma_{i1}}\right)^{\gamma_{i2}}$ ko‘rinishda izlab,

$\gamma_{i1}=\dfrac{p_i+2}{p_i+1}$, $\gamma_{i2}=\dfrac{p_i+1}{m_i(p_i+1)-1}$, $a_i>0$, $b_i=\dfrac{1-m_i(p_i+1)}{m_i(p_i+2)}\beta_i^{\frac{1}{p_i+1}}$, $y_+=\max(0,y)$,

natija olinadi.

**Teorema1.** Aytaylik $\min\{N\beta_i-\alpha_i\}>0$ shart bajarilsin, u holda (1) masalaning yechimi global bo‘ladi.

**Isbot**. Quyidagi $\bar{v}_i(\eta_i)$ yechimni

$$\begin{cases} \eta^{1-N}\left(\eta^{N-1}\left|\left(v_i^{m_i}\right)'\right|^{p_i}\left(v_i^{m_i}\right)'\right)'(\eta_i)+\alpha_i v_i(\eta_i)+\eta_i\beta_i v_i'(\eta_i)\le 0, \\ -\left|\left(v_i^{m_i}\right)'\right|^{p_i}\dfrac{\partial v_i^{m_i}}{\partial \eta_1}\ge v_{i+1}^{q_i}(\eta_{i+1}), \qquad i=1,2,...,k. \end{cases} \tag{11}$$

(11) ga qo‘yish orqali quyidagi ko‘rinishga kelamiz:

$$\begin{cases} v_i(\eta_i)\left(\alpha_i-\beta_i N\right)\le 0, \\ \left(\dfrac{(p_i+2)m_i}{1-m_i(p_i+1)}b_i\right)^{p_i+1}(a_i+b_i)^{-\frac{p+1}{1-m_i(p_i+1)}}\ge(a_{i+1}+b_{i+1})^{\frac{p_{i+1}+1}{1-m_{i+1}(p_{i+1}+1)}q_i}, \end{cases} \tag{12}$$

(12) tengsizlik teorema sharti bajarilganda (1) masalani yechimini global bo‘lishini ko‘rsatadi.

**Izoh 1.** 1-teoremaga ko‘ra (1) masala uchun Fudjita tipidagi kritik eksponentalar qiymatlari quyidagicha bo‘ladi: $\min\{N\beta_i-\alpha_i\}=0$.

Belgilashlar kiritamiz:

$$q_{i0}=\frac{m_i(p_i+1)}{\alpha_1\left(m_i(p_i+1)-1\right)},\ q_{ic}=\frac{(p_i+1)\left(Nm_i+1\right)}{\alpha_1((p_i+2)-N(1-m_i(p_i+1)))},$$

bu yerda $q_{i0}$ global yechim mavjudlik kritik eksponentasi, $q_{ic}$ Fudjita tipidagi kritik eksponenta.

**Teorema2.** Agar quyidagi shart $q_{i0}<q_i<q_{ic}$ bajarilsa, u holda (1) masalaning har qanday notrival yechim chekli vaqt ichida chegaralanmagan (blow up) bo‘ladi.

**Isbot:** Teoremani isbotlash uchun yechimni quyidagicha olamiz.

$$u_{Bi}(x,t)=(\tau+t)^{-\frac{N}{1+(m_iN+1)(p_i+1)-N}}v_i(\eta_i)\ ,\eta_i=\left|\xi_i\right|,\ \xi_i=(1+x)(\tau+t)^{-\frac{1}{1+(m_iN+1)(p_i+1)-N}}, \tag{13}$$

bu yerda $a_i>0$, $\bar{v}_i(\eta_i)=\left(a_i+b_i\eta_i^{\gamma_1}\right)^{\gamma_2}$, $b_i=\dfrac{1-m_i(p_i+1)}{m_i(p_i+2)}\beta_i^{\frac{1}{p_i+1}}$.

$\eta_i\in\{\eta>0\,|\,v_i(\eta_i)\ge 0\}$ uchun quyidagi shartlarni qanoatlantiradigan $-\left|\left(v_i^{m_i}\right)'\right|^{p_i}\left.\dfrac{\partial v_i^{m_i}}{\partial \eta_1}\right|_{\eta_1=0}=0$ va

$$\eta^{1-N}\frac{d}{d\eta_i}\left(\eta^{N-1}\left|\frac{dv^{m_i}{}_i}{d\eta_i}\right|^{p_i}\frac{dv_i^{m_i}}{d\eta_i}\right)+\frac{N}{1+(m_iN+1)(p_i+1)-N}v_i(\eta_i)+$$
$$+\frac{1}{1+(m_iN+1)(p_i+1)-N}\eta_i\frac{dv_i}{d\eta_i}\le 0, \tag{14}$$

(1) masalaning umumlashgan yechimining xossalaridan foydalanib, $t_0>0$ bo‘lsa, $u_i(0,t_0)>0$ ekanligini ko‘ramiz. Shunday qilib $\tau>0$ va $a_i>0$ mavjud bo‘ladiki, $u_i(x,t_0)\ge u_{Bi}(x,t_0),\ x\in R_+^N$.

Hisoblashlar shuni ko‘rsatadiki, $u_{Bi}(x,t)$ (1) masalaning $R_0^N\times(t_0,+\infty)$ quyi yechimi hisoblanadi. Taqqoslash prinsipi ko‘ra quyidagini hosil qilamiz $u_i(x,t)\ge u_{Bi}(x,t),(x,t)\in R_+^N\times(t_0,+\infty)$. Endi $t_*\ge t_0$ va $T$ yetarlicha katta bo‘lsin, shunda

$$u_{Bi}(x,t_*)\ge\overline{u}_i(x,0),\quad x\in R_+^N \tag{15}$$

bu yerda $\overline{u}_i(x,t)$ (9) da berilgan.

Hisoblashlardan so‘ng, (15) dan quyidagiga keladi:

$$\begin{aligned}&(\tau+t_*)^{-\frac{N}{1+(m_iN+1)(p_i+1)-N}}\ ?\ T^{-\frac{\alpha_1q_i(p_i+2)-(p_i+1)}{(m_i+1)(p_i+1)}},\\ &(\tau+t_*)^{-\frac{1}{1+(m_iN+1)(p_i+1)-N}}=T^{-\frac{\alpha_1q_i(1-m_i(p_i+1))+m_i(p_i+1)}{(m_i+1)(p_i+1)}}.\end{aligned} \tag{16}$$

Shunday qilib, agar $q_i<\dfrac{(p_i+1)(Nm_i+1)}{\alpha_1((p_i+2)-N(1-m_i(p_1+1))}$ shart bajarilsa, unda

$$\frac{N(\alpha_1q_i(1-m_i(p_i+1))+m_i(p_i+1))}{(1+(m_iN+1)(p_i+1)-N)(m_i+1)(p_i+1)}<\frac{\alpha_1q_i(p_i+2)-(p_i+1)}{(1+(m_iN+1)(p_i+1)-N)(m_i+1)(p_i+1)},$$

bo‘ladi. Shuning uchun yetarlicha katta $t_*$ va $T$ uchun (16) tengsizliklar o‘rinli bo‘ladi. Demak, $u_i(x,t_*)\ge u_{Bi}(x,t_*),\ \ x\in R_+^N$ taqqoslash prinsipi bilan birgalikda $u_i(x,t)$ funksiyasi chekli vaqtda ichida chegaralanmagan (blow -up) bo‘ladi.

**Teorema 3.** Agar $p_i<\dfrac{1}{m_i}-1$ bo‘lsa, u holda (1) masalaning yechimi quyidagi asimptotikaga ega bo‘ladi:

$$\overline{u}_i(x,t)=(\tau+t)^{-\alpha_i}\left(a_i+b_i\eta_i^{\frac{p_i+2}{p_i+1}}\right)^{\frac{p_i+1}{m_i(p_i+1)-1}}, \tag{17}$$

bu yerda $a_i>0,\ b_i=\dfrac{1-m_i(p_i+1)}{m_i(p_i+2)}\beta_i^{\frac{1}{p_i+1}}$, $y_+=\max(0,y)$ bo‘lganda.

**Isbot.** (1) masalani asimtotik xatti-harakatiga ega bo‘lish uchun, (10)-avtomodel tenglamani yechimini $\overline{u}_i(x,t)$ ko‘rinishda qidiramiz.

$$\begin{aligned}&\eta^{N-1}\left|\frac{d\overline{h}_i^{m_i}}{d\eta_i}\right|^{p_i}\frac{d\overline{h}_i^{m_i}}{d\eta_i}=b_i^{\frac{p_i+1}{p_i+2}(p_i+2-N)}\left|\frac{p_i+2}{p_i+1}\right|^{p_i+1}\left(e^{\tau_i}-a_i\right)^{\frac{N(p_i+1)}{p_i+2}}\overline{h}_i\cdot Kw_i;\\ &\eta^{1-N}\frac{d}{d\eta_i}\left(\eta^{N-1}\left|\frac{d\overline{h}_i^{m_i}}{d\eta_i}\right|^{p_i}\frac{d\overline{h}_i^{m_i}}{d\eta_i}\right)=b_i^{p_i+1}\left|\frac{p_i+2}{p_i+1}\right|^{p_i+2}\left(1-a_i\cdot e^{-\tau_i}\right)\overline{h}_i\cdot K^*w_i;\\ &\alpha_i\overline{h}(\eta_i)+\eta_i\beta_i\frac{d\overline{h}_i}{d\eta_i}=\overline{h}_i\cdot\left(\frac{\beta_i(p_i+2)}{p_i+1}\left(1-a_i\cdot e^{-\tau_i}\right)w_i^*+\alpha_iw_i\right);\end{aligned} \tag{18}$$

(18) tengliklarni (10)- tenglamaga olib borib qo‘yib,

$$K^* w_i + \left|\frac{p_i+1}{p_i+2}\right|^{p_i+2} b_i^{-(p_i+1)} \left[\frac{\beta_i(p_i+2)}{p_i+1} w_i^* + \frac{1}{1-a_i e^{-\tau_i}} \alpha_i w_i\right] = 0,$$

bu yerda $Kw_i = \left|\frac{dw_i^{m_i}}{d\tau_i} + \frac{m_i(p_i+1)}{m_i(p_i+1)-1} w_i^{m_i}\right|^{p_i} \left(\frac{dw_i^{m_i}}{d\tau_i} + \frac{m_i(p_i+1)}{m_i(p_i+1)-1} w_i^{m_i}\right),$

$$K^* w_i = \frac{dK_i}{d\tau_i} + \left(\frac{N(p_i+1)}{p_i+2} \cdot \frac{1}{1-a_i e^{-\tau_i}} + \frac{p_i+1}{m_i(p_i+1)-1}\right) K_i, \qquad w_i^* = \frac{dw_i}{d\tau_i} + \frac{p_i+1}{m_i(p_i+1)-1} w_i.$$

Agar $\tau_i \to \infty$ bo‘lsa, u holda $\frac{1}{1-a_i e^{-\tau_i}} \to 1, \quad w_i^* \to \frac{p_i+1}{m_i(p_i+1)-1} w_i^0$ bo‘ladi, undan

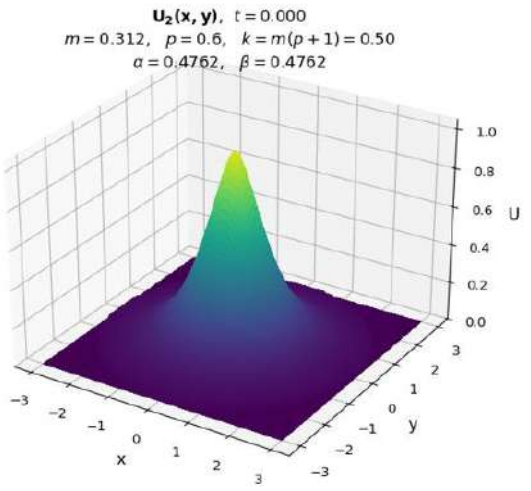
$$K^* w_i \to \frac{|m_i(p_i+1)|^{p_i+1} \cdot (p_i+1)}{|m_i(p_i+1)-1|^{p_i} (m_i(p_i+1)-1)} \left(\frac{N}{p_i+2} + \frac{1}{m_i(p_i+1)-1}\right) \left|\overset{0}{w}_i\right|^{m_i(p_i+1)}$$ kelib chiqadi.

Natijada $\overset{0}{w}_i = \beta_i \left(p_i + 2 + N(m_i(p_i+1)-1)\right)^{\frac{1}{1-m_i(p_i+1)}}$ ekanligi isbotlanadi.

Ushbu ishda bir jinsli bo‘lmagan muhitdagi sekin diffuziya holi uchun chiziqsiz issiqlik tarqalish masalasi yechimlarining vaqt bo‘yicha global bo‘lish shartlari, avtomodel yechimlarning asimptotikalari tadqiq etiladi.

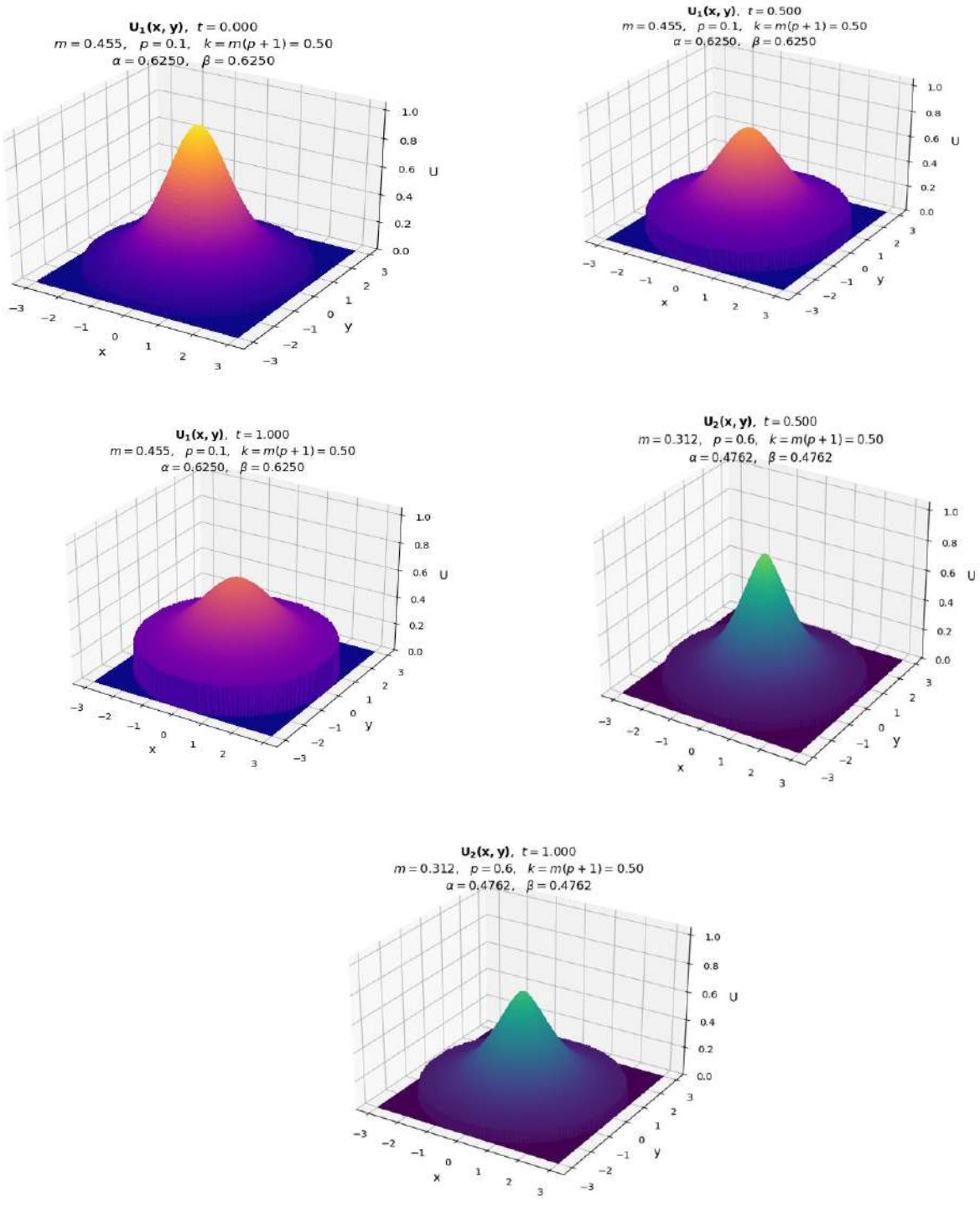
**Hisoblash eksperimenti va vizualizatsiya natijalari**

Qo‘yilgan (1) masala saqlanish qonunlariga muvofiq Balans usuli asosida ko‘p o‘lchovli issiqlik tarqalish masalasi uchun oshkormas ayirmali sxemalar qurilgan [5; 4-bob]. Natijada hosil bo‘lgan nochiziqli uch dioganalli algebraik tenglamalar sistemasini chiziqlashtirish uchun oddiy iteratsiya usulidan foydalanib, chiziqlashtirilib, haydash (progonka) usuli yordamida yechildi. Bunda, iteratsion jarayon uchun boshlang‘ich yaqinlashish sifatida (9), (17) dan foydalanilgan. Quyida $k = 3$ da olingan sonli natijalar keltirilgan.



**1-rasm. (1) masalaning sonli yechimi.**

1-rasmda tez diffuziya holatiga mos keladigan vaqtning turli momentlari uchun (1) masalaning $\min\left\{N\beta_i-\alpha_i\right\}>0,\quad p_i<\dfrac{1}{m_i}-1,\ 0<m_i<1,\ q_i>0$ bo‘lgandagi sonli yechimlarining grafiklari keltirilgan. (1) masalani sonli yechishda iteratsion jarayon uchun boshlang‘ich yaqinlashish sifatida (9), (17) lar olindi. Ma’lumki, $p_i<\dfrac{1}{m_i}-1$ tez diffuziya holi bo‘lib, tarqalish cheksiz tezlikda sodir bo‘ladi. Iteratsiya 5-6 qadamda yaqinlashdi. Xatolik $\varepsilon=10^{-5}$ ga teng.




**2-rasm. (1) masalaning sonli yechimi.**

2-rasmda (1) masalani sonli yechish natijalari tasvirlangan bo‘lib, ularda issiqlik tarqalishi cheksiz tezlik bilan sodir bo‘ladi.

**Xulosa va takliflar.** Maqolada chiziqsiz chegaraviy shartlar bilan bog‘langan, tez diffuziya jarayonini tavsiflovchi parabolik tipdagi tenglamalar sistemasining sifat xossalari avtomodel yondoshuv asosida tadqiq etildi. Avtomodel yondashuv va yechimlarni solishitirish prinsipi asosida yechimlarning vaqt bo‘yicha global bo‘lishi va chegaralanmagan yechimlarning hosil bo‘lish shartlari topildi. Hisoblash eksperimentida asimptotik formulalardan boshlang‘ich yaqinlashish sifatida foydalanib, sonli yechimlar turli parametrik qiymatlarda tez diffuziya jarayoniga xos bo‘lgan issiqlik tarqalishi ko‘rsatildi.

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