UO‘Q 517.98

# $R^2$ DA ANIQLANGAN NOCHIZIQLI OPERATORLARNING QO‘ZG‘ALMAS NUQTALARI VA ULARNING BA’ZI TATBIQLARI

***Nodirov Shohruh Dilmurodovich,***
*Qarshi davlat universiteti*
*Matematik analiz va differensial*
*tenglamalar kafedrasi mudiri*
*shoh0809@mail.ru*
***Quldosheva Sojida Orziqul qizi,***
*Qarshi davlat universiteti*
*magistranti*
*quldoshvasojida@gmail.com*

***Annotatsiya.*** *Mazkur ishda* $\boldsymbol{R^2}$ *da aniqlangan musbat koeffitsiyentli k-tartibli bir jinsli nochiziqli operatorlarning qo‘zg‘almas nuqtalari tadqiq qilingan. Qaralayotgan masala (k+1)-tartibli ko‘phadning musbat ildizlarini topishga keltirilgan hamda bu orqali qaralayotgan nochiziqli operatorning* $\boldsymbol{R^2}$ *da musbat qo‘zg‘almas nuqtasi mavjudligi isbotlangan. Gammershteyn tipiga mansub nochiziqli integral tenglamalarning yechimini topish esa qaralayotgan nochiziqli operatorning qo‘zg‘almas nuqtasiga keltirilishi ko‘rsatilgan. k=2, k=3 va k=4 holatlar uchun yechimning yagonaligi isbotlangan hamda yechimning aniq ko‘rinishi keltirilgan. Olingan natijalar k-tartibli Keli daraxtida aniqlangan translatsion-invariant Gibbs o‘lchovlariga tatbiq qilingan.*

***Kalit so‘zlar:*** *nochiziqli operator, qo‘zg‘almas nuqta, ko‘phad, Gammershteyn tipidagi integral tenglama, yadro, ajraluvchi yadro, simmetrik yadro, trivial bo‘lmagan yechim, binomial koeffitsiyent, Dekard qoidasi, translatsion-invariant, Gibbs o‘lchovi, Keli daraxti.*

# НЕПОДВИЖНЫЕ ТОЧКИ НЕЛИНЕЙНЫХ ОПЕРАТОРОВ, ОПРЕДЕЛЁННЫХ В $R^2$, И НЕКОТОРЫЕ ИХ ПРИЛОЖЕНИЯ

***Аннотация.*** *В настоящей работе исследованы неподвижные точки однородных нелинейных операторов порядка k с положительными коэффициентами, определённых в* $\boldsymbol{R^2}$*. Рассматриваемая задача сведена к нахождению положительных корней многочлена степени (k+1), что позволило доказать существование положительной неподвижной точки исследуемого нелинейного оператора в* $\boldsymbol{R^2}$*. Показано, что задача нахождения решений нелинейных интегральных уравнений типа Гаммерштейна сводится к задаче о неподвижных точках рассматриваемого нелинейного оператора. Для случаев k=2, k=3 и k=4 доказана единственность решения, и получено его явное представление. Полученные результаты применены к исследованию трансляционно-инвариантных мер Гиббса на дереве Кэли порядка k.*

***Ключевые слова:*** *нелинейный оператор, неподвижная точка, многочлен, нелинейное интегральное уравнение типа Гаммерштейна, ядро, вырожденное ядро, симметрическое ядро, нетривиальное решение, биномиальный коэффициент, правило Декарта, трансляционно-инвариантный, мера Гиббса, дерево Кэли.*

# FIXED POINTS OF NONLINEAR OPERATORS DEFINED ON $R^2$ AND SOME OF THEIR APPLICATIONS

***Abstract.*** *This paper investigates the fixed points of homogeneous nonlinear operators of order k with positive coefficients defined in* $\boldsymbol{R^2}$*. The considered problem is reduced to finding the positive roots of a polynomial of degree (k+1), which makes it possible to prove the existence of a positive fixed point of the corresponding nonlinear operator in* $\boldsymbol{R^2}$*. It is further shown that the problem of finding solutions to Hammerstein-type nonlinear integral equations can be reduced to the fixed-point problem for the considered nonlinear operator. For the cases k=2, k=3 and k=4 the uniqueness of the solution is established, and its explicit form is obtained. The obtained results are applied to the study of translation-invariant Gibbs measures on the Cayley tree of order k.*

***Keywords:*** *nonlinear operator, fixed point, polynomial, Hammerstein-type nonlinear integral equation, kernel, degenerate kernel, symmetric kernel, nontrivial solution, binomial coefficient, Descartes' rule of signs, translation-invariant, Gibbs measure, Cayley tree.*

**Kirish.** Ma'lumki, hozirgi kunda turli sohalardagi ko'plab zamonaviy tadqiqotlar matematikada uchraydigan nochiziqli operatorlar nazariyasini tadqiq etishga keltirilmoqda. Bu esa o'z navbatida mazkur sohaga bo'lgan qiziqishni yanada kuchaytirmoqda. Nochiziqli operatorlarni tadqiq etish uchun umumiy nazariyaning mavjud emasligi, unga doir masalalarni yechishda alohida yondashuvlar ishlab chiqishni taqozo qilmoqda. Ushbu yo'nalishdagi asosiy masalalardan biri bu qo'zg'almas nuqta masalasi bo'lib, uning mavjudligi, yagonaligi, qo'zg'almas nuqtalarning soni hamda uning ko'rinishini topish muhim hisoblanadi.

Dastlab 2010-yilda Keli daraxtidagi translatsion-invariant Gibbs o'lchovlarini tadqiq qilish masalasi nochiziqli integral tenglamalarni yechish masalasiga kelishi isbotlandi [1]. Shundan so'ng [2-3] ishlarda mazkur masalalar Gammershteyn tipiga mansub nochiziqli integral tenglamasini musbat qo'zg'almas nuqtalarini topish masalasiga keltirildi. Qaralayotgan Gammershteyn integral tenglamasining yadrosi ajraluvchan holatda bir qancha tadqiqotlar olib borilgan hamda musbat qo'zg'almas nuqtalari soni uchun kerakli natijalar olingan ([4-8] ishlarga qarang). Bu sohada ko'plab tatqiqotlar olib borilganiga qaramasdan, yadro ajraluvchi bo'lgan holatda ixtiyoriy darajali Gammershteyn integral tenglamasini musbat qo'zg'almas nuqtalari soni haqida yetarli natijalar mavjud emas emas. Shundan so'ng yadroni ajraluvchi simmetrik holatda qaralsa qanday natijalar olish mumkinligi qiziqish uyg'otdi [8]. Mazkur ishni shu kabi masalalarning davomi sifatida qarash mumkin.

## $\mathbf{R^2}$ da aniqlangan nochiziqli operatorlarning musbat qo'zg'almas nuqtalari

Biz $Q_k: R^2 \longrightarrow R^2$ nochiziqli operatorni quyidagicha aniqlaylik:

$$Q_k(x,y)=\left(\sum_{i=0}^{k} C_k^i a_i x^{k-i}y^i\,, a_{k+1}x^k+\sum_{i=1}^{k} C_k^i a_{i-1}x^{k-i}y^i\right), \quad (k\in\mathbb{N}\backslash\{1\})$$

bu yerda $C_k^i=\frac{k!}{(k-i)!i!}$ (Binomial koeffitsiyent) va barcha $i\in\{0,1,\dots,k+1\}$ lar uchun $a_i>0$.

$R^2$ da quyidagi to'plamni qaraylik:

$$R_{>}^2=\{(x,y): x>0, y>0\}.$$

Ushbu

$$P_{k+1}(\xi)=a_k\xi^{k+1}+\sum_{j=1}^{k} C_k^j\left(\frac{k-2j+1}{k-j+1}\right)a_{k-j}\xi^{k-j+1}-a_{k+1}$$

$k+1$ darajali ko'phadni aniqlaylik.

**2.1-lemma.** *$Q_k$-operator*

$$\left(\frac{1}{\left(\sum_{i=0}^{k} C_k^i a_i \xi_0^i\right)^{\frac{1}{k-1}}}, \frac{\xi_0}{\left(\sum_{i=0}^{k} C_k^i a_i \xi_0^i\right)^{\frac{1}{k-1}}}\right)\in R_{>}^2$$

*qo'zg'almas nuqtaga ega bo'lishi uchun*

$$P_{k+1}(\xi)=0$$

*tenglama $\xi_0>0$ yechimga ega bo'lishi zarur va yetarli.*

**Isbot.** *Zarurligi*. Aytaylik $Q_k$ operator

$$\omega_0=\left(\frac{1}{\left(\sum_{i=0}^{k} C_k^i a_i \xi_0^i\right)^{\frac{1}{k-1}}}, \frac{\xi_0}{\left(\sum_{i=0}^{k} C_k^i a_i \xi_0^i\right)^{\frac{1}{k-1}}}\right)$$

qo'zg'almas nuqtaga ega bo'lsin. U holda quyidagi tenglik hosil bo'ladi:

$$\xi_0=\frac{a_{k+1}+\sum_{i=1}^{k} C_k^i a_{i-1}\xi_0^i}{\sum_{i=0}^{k} C_k^i a_i \xi_0^i}.$$

Oxirgi tenglikni soddalashtirib quyidagi tenglamani hosil qilamiz:

$$a_k\xi_0^{k+1}+\sum_{i=1}^{k} C_k^{i-1}\left(1-\frac{k-i+1}{i}\right)a_{i-1}\xi_0^i-a_{k+1}=0.$$

Qulaylik uchun ushbu

$$i=k-j+1$$

belgilashdan hamda

$$C_k^{k-j} = C_k^j$$

tenglikdan foydalansak, ko‘phad tenglamasi quyidagi ko‘rinishga keladi:

$$P_{k+1}(\xi) := a_k\xi_0^{k+1} + \sum_{j=1}^{k} C_k^j \left(\frac{k-2j+1}{k-j+1}\right) a_{k-j}\xi_0^{k-j+1} - a_{k+1} = 0.$$

*Yetarliligi.* Aytaylik $P_{k+1}(\xi)$ ko‘phad $\xi_0$ musbat ildizga ega bo‘lsin. U holda

$$Q_k(x_0, \xi_0 x_0) = (x_0, \xi_0 x_0)$$

tenglik o‘rinli bo‘ladi. Bu yerda

$$x_0 = \frac{1}{(\sum_{i=0}^{k} C_k^i a_i \xi_0^i)^{\frac{1}{k-1}}}.$$

1-lemma isbotlandi.

**2.2-lemma.** *$P_{k+1}(\xi)$ ko‘phad kamida bitta musbat ildizga ega.*

**Isboti.** Haqiqatdan ham $P_{k+1}(0) = -a_k$, $P_{k+1}(+\infty) = +\infty$ munosabatga ko‘ra $P_{k+1}(\xi)$ ko‘phad $[0, +\infty)$ oraliqda yechimga ega bo‘ladi.

## 3. Gammershteyn tipiga mansub integral tenglamalarning musbat yechimlari

Gammershteyn integral tenglamasi integral tenglamalar nazariyasida muhim o‘rin tutadi. Bu tenglama quyidagi ko‘rinishda ifodalanadi:

$$f(t) = v(t) + \lambda \int_a^b K(t,u)F(u, f(u))du$$

bu yerda $f(u)$ -noma’lum funksiya, $v(t)$-ma’lum funksiya, $K(t,u)$ - yadro funksiyasi, $F(u, f(u))$-nochiziqli funksiya, $\lambda$-parametr, *[a,b]*-integral olinadigan oraliq.

Birinchi tur Gammershteyn tenglamasi quyidagi ko‘rinishda bo‘ladi:

$$\int_a^b K(t,u)F(u, f(u))du = f(t) \quad (1)$$

Biz har bir $k \in \mathbb{N}\backslash\{1\}$ uchun nochiziqli funksiyani $F\big(u, f(u)\big) = f^k(u)$ holatda, integrallash oralig‘ini [0,1] ko‘rinishda aniqlasak, u holda (1) tenglama

$$\int_0^1 K(t,u)f^k(u)du = f(t) \quad (2)$$

ko‘rinishga keladi.

Qulaylik uchun $[0,1]$ dagi uzluksiz funksiyalar fazosi $C[0,1]$ da

$$C_>[0,1] = \{f(t) > 0 : f(t) \in C[0,1]\}$$

sinfni aniqlab olaylik.

Biz $K(t,u)$ yadroni

$$K(t,u) = \varphi(t)\psi(u) + \varphi(u)\psi(t), \quad \varphi(t), \psi(t) \in C_>[0,1]$$

kabi (ya’ni ajraluvchan simmetrik ($K(t,u) = K(u,t)$) ko‘rinishida) qaraylik.

U holda (2) tenglama

$$\int_0^1 (\varphi(t)\psi(u) + \varphi(u)\psi(t))f^k(u)du = f(t) \quad (3)$$

ko‘rinishga keladi. Ta’kidlash joizki, biz (3) tenglamaning trivial bo‘lmagan musbat yechimlarini qidiramiz.

Ma’lumki (3) tenglamaning yechimi

$$f(t) = \varphi(t)x + \psi(t)y \quad (4)$$

ko‘rinishda bo‘ladi, bu yerda

$$x = \int_0^1 \psi(u)f^k(u)du > 0,\ y = \int_0^1 \varphi(u)f^k(u)du > 0. \quad (5)$$

(5) belgilashga (4) yechimni olib borib qo‘yamiz, ya’ni

$$x = \int_0^1 \psi(u)(\varphi(u)x + \psi(u)y)^k\, du,\ y = \int_0^1 \varphi(u)(\varphi(u)x + \psi(u)y)^k du.$$

Biz

$$a_i = \int_0^1 \varphi^{k-i}(u)\psi^{i+1}(u)du > 0, \quad i = \overline{0,1,\dots,k},\ a_{k+1} = \int_0^1 \varphi^{k+1}(u)du > 0 \quad (6)$$

kabi belgilashlarni amalga oshiramiz. Natijada esa,

$$x=\sum_{i=0}^{k} C_k^i a_i x^{k-i} y^i \ , \quad y=a_{k+1}x^k+\sum_{i=1}^{k} C_k^i a_{i-1} x^{k-i} y^i,$$

tenglamalar sistemasini hosil qilamiz. Oxirgi sistema esa $Q_k$ operatorning $R_>^2$ dagi musbat yechimini topish masalasiga kelishini ko‘rishimiz mumkin.

Aytaylik, $P_{k+1}(\xi)$ ko‘phadning koeffitsiyentlari (5) tenglik yordamida aniqlangan bo‘lsin. U holda quyidagi teorema o‘rinli.

**3.1-teorema.** *Har qanday $k \in \{2,3,4\}$ uchun (3) integral tenglama yagona*

$$f(t)=\frac{\varphi(t)+\psi(t)\xi_0}{\left(\sum_{i=0}^{k} C_k^i \xi_0^i \int_0^1 \varphi^{k-i}(u)\psi^{i+1}(u)du\right)^{\frac{1}{k-1}}}, t \in [0,1]$$

*musbat yechimga ega bo‘ladi. Bu yerda $\xi_0$, $P_{k+1}(\xi)$ ko‘phadning musbat ildizi.*

**Isbot.** Ma’lumki (4) va (6) belgilashlarga ko‘ra hamda 2.1-lemmaga ko‘ra tenglama yechimining ko‘rinishini hosil qilish murakkablik tug‘dirmaydi. Qolaversa 2.1-lemmaga ko‘ra $P_3(\xi), P_4(\xi), P_5(\xi)$ ko‘phadlarning musbat ildizini tadqiq etish yetarli. 1.2-lemmaga ko‘ra qaralayotgan ko‘phadlarning musbat ildizlari mavjud. Shu sababli musbat ildizining yagona ekanini isbotlaymiz.

**$k=2$ holat.**

$$P_3(\xi)=a_2\xi^3+a_1\xi^2-a_0\xi-a_3$$

3-darajali ko‘phadni qaraylik. Ko‘rinib turibdiki $P_3(\xi)$ ko‘phad funksiyaning $O\xi$ o‘qining musbat yo‘nalishi bilan kesishish nuqtalari, uning musbat ildizlarini tashkil etadi. Shu sababli, $\eta=P_3(\xi)$ funksiyaning grafigini tahlil qilamiz. $\eta=P_3(\xi)$ funksiyaning kritik nuqtalarini topamiz, ya’ni

$$P_3'(\xi)=3a_2\xi^2+2a_1\xi-a_0=0$$

tenglamani yechamiz. Natijada

$$\xi_1=\frac{-2a_1-\sqrt{4(a_1)^2+12a_2a_0}}{6a_2}, \xi_2=\frac{-2a_1+\sqrt{4(a_1)^2+12a_2a_0}}{6a_2}$$

ildizlarni topamiz. Aniqlanishiga ko‘ra $a_0, a_1$ va $a_2$ larning qat’iy musbat ekanidan

$$\xi_1<0, \xi_2>0$$

munosabatlar o‘rinli ekanini ko‘rish qiyin emas. Aniqlanishiga ko‘ra $\eta=P_3(\xi)$ funksiyaning grafigi $(-\infty,\xi_1]$ oraliqda o‘suvchi, $(\xi_1,\xi_2)$ oraliqda kamayuvchi, $[\xi_2+\infty)$ oraliqda esa o‘suvchi bo‘ladi. $P_3(0)=-a_3<0$ munosabatning hisobiga esa, $\eta=P_3(\xi)$ funksiyaning grafigi $O\xi$ o‘qining musbat yo‘nalishi bilan yagona nuqtada kesishadi. Demak, $P_3(\xi)$ ko‘phad yagona musbat ildizga ega.

**$k=3$ holat.** Ushbu

$$P_4(\xi)=a_3\xi^4+2a_2\xi^3-2a_0\xi-a_4$$

ko‘phadning musbat ildizini tahlil qilamiz. Aniqlanishiga ko‘ra $a_3$, $a_2$, $a_0$ va $a_4$ lar musbat sonlar. $P_4(\xi)$ ko‘phad funksiyaning $O\xi$ o‘qining musbat yo‘nalishi bilan kesishish nuqtalarini tahlil qilamiz. Buning uchun

$$P_4'(\xi)=4a_3\xi^3+6a_2\xi^2-2a_0$$

ko‘phadning musbat ildizlarini tahlil qilishimiz kerak. Ma’lumki,

$$P_4'(-\infty)=-\infty, P_4'(0)=-2a_0<0, P_4'(+\infty)=+\infty,$$

munosabatlar o‘rinli bo‘ladi. Uni tahlil qilish uchun yana bir marta hosila olib nollarini topamiz, ya’ni

$$P_4''(\xi)=12a_3\xi^2+12a_2\xi=0.$$

$P_4''(\xi)$ funksiyaning ildizlari bo‘lgan $\xi_1=-\frac{a_2}{a_3}<0$ , $\xi_2=0$ nuqtalar $P_4'(\xi)$ funksiyaning kritik nuqtalarini tashkil etadi.

$P_4'(\xi)$ funksiyaning grafigi $(-\infty,\xi_1]$ oraliqda o‘suvchi, $(\xi_1,0]$ oraliqda kamayuvchi, $(0,+\infty)$ oraliqda esa o‘suvchi bo‘lib, $P_4'(0)<0$ ekanligidan, $P_4'(\xi)$ ko‘phadning grafigi $(0,+\infty)$ ni yagona nuqtada kesib o‘tishi kelib chiqadi.

Ko‘rinib turibdiki,

$$P_4'(\xi)=4a_3\xi^3+6a_2\xi^2-2a_0$$

funksiyaning yagona $\boldsymbol{\xi_0>0}$ musbat ildizga ega ekanidan, $\boldsymbol{P_4(\xi)}$ ko‘phadning grafigi $[\boldsymbol{\xi_0},+\infty)$ da o‘suvchi hamda $\min\limits_{[\mathbf{0},+\infty)} \boldsymbol{P_4(\xi)}=\boldsymbol{P_4(\xi_0)}$ ($\boldsymbol{P_4}(\mathbf{0})<\mathbf{0}$ ekanligidan) munosabat o‘rinli bo‘lishini ko‘rish qiyin emas. Demak, $\boldsymbol{P_4(\xi)}$ ko‘phad $[\boldsymbol{\xi_0},+\infty)$ da yagona musbat ildizga ega bo‘ladi.

**$k=4$ holat.** Ushbu holatda beshinchi darajali quyidagi ko‘phad hosil bo‘ladi:

$$P_5(\xi)=a_4\xi^5+3a_3\xi^4+2a_2\xi^3-2a_1\xi^2-3a_0\xi-a_5.$$

Ma'lumki bu yerda $a_4$, $a_3$, $a_2$, $a_1$, $a_0$ va $a_5$ lar musbat sonlar. $P_5(\xi)$ ko'phad funksiyaning $O\xi$ o'qining musbat yo'nalishi bilan kesishish nuqtalarini qidiraylik. Endi, $\eta = P_5(\xi)$ funksiyaning kritik nuqtalarini topamiz,

$$P_5'(\xi) = 5a_4\xi^4 + 12a_3\xi^3 + 6a_2\xi^2 - 4a_1\xi - 3a_0,$$
$$P_5'(-\infty) = +\infty, P_5'(0) = -3a_0 < 0, P_5'(+\infty) = +\infty$$
$$P_5''(\xi) = 20a_4\xi^3 + 36a_3\xi^2 + 12a_2\xi - 4a_1$$
$$P_5''(-\infty) = -\infty, P_5''(0) = -4a_1 < 0, P_5''(+\infty) = +\infty$$
$$P_5'''(\xi) = 60a_4\xi^2 + 72a_3\xi + 12a_2$$

$P_5''(\xi)$ ko'phad grafigi $[0,+\infty)$ da o'suvchi (chunki har qanday $\xi > 0$ da $P_5'''(\xi) = 60a_4\xi^2 + 72a_3\xi + 12a_2 > 0$ ) bo'ladi. Shu sababli $P_5''(\xi)$ ko'phad $[0,+\infty)$ da yagona $\xi_0 > 0$ ildizga ega bo'ladi. Bundan esa $\min\limits_{[0,+\infty)} P_5'(\xi) = P_5'(\xi_0)$ munosabat o'rinli bo'lib $[\xi_0,+\infty)$ oraliqda o'suvchidir. Demak, $P_5'(\xi)$ ko'phad $[\xi_0,+\infty)$ oraliqda yagona $\xi_1$ musbat ildizga ega. $P_5'(0) < 0$ munosabatga ko'ra $P_5(\xi)$ uchun $\min\limits_{[0,+\infty)} P_5(\xi) = P_5(\xi_1) < 0$ bo'lib, $[\xi_1,+\infty)$ da o'suvchi ekanligidan shu oraliqda yagona musbat ildizga ega ekanligi kelib chiqadi**.** 3.1-teorema isbotlandi.

**Keli daraxtida aniqlangan translatsion-invariant Gibbs o'lchovlari**

Aytaylik, $k$ −tartibli $\Gamma_k = (V, L)$ Keli daraxti berilgan bo'lsin. Bu bir jinsli va siklsiz cheksiz graf bo'lib, uning har bir uchidan aynan $k+1$ ta shox (qirra) chiqadi. Bu yerda $V$ grafning uchlari, $L$ esa qirralari to'plami. $x$ va $y$ uchlarini tutashtiruvchi yagona $l \in L$ qirra mavjud bo'lsa, u holda ular yaqin qo'shnilar deyiladi hamda $l = \langle x, y\rangle$ ko'rinishida belgilanadi. Berilgan $\langle x, x_1\rangle, \langle x_1, x_2\rangle, \dots, \langle x_{d-1}, y\rangle$ yaqin qo'shnilar to'plami $x$ dan $y$ ga boruvchi yo'lak deb ataladi. Keli daraxtida $d(x, y)$ masofa $x$ dan $y$ ga boruvchi eng qisqa yo'laklar soniga teng, ya'ni

$$d(x, y) = min\{d: \exists x = x_0, x_1, \dots, x_{d-1}, x_d = y \in V, bu\ yerda\ \langle x_0x_1\rangle, \dots, \langle x_{d-1}x_d\rangle\}.$$

$\Gamma_k$ Keli daraxtida markazi $x_0$ nuqtada, radiusi $l = 1,2, \dots$ bo'lgan $W_l$ sfera va $V_l$ sharni quyidagicha aniqlaymiz:

$$W_l = \{x \in V: d(x, x^0) = l\}, V_l = \{x \in V: d(x, x^0) \le l\},$$
$$L_n = \{\langle x, y\rangle \in L: x, y \in V_n\}.$$

Agar $x^0$ dan $y$ ga o'tuvchi yo'lak $x$ orqali o'tsa, u holda $x < y$ kabi belgilanadi. Agar $y > x$ va $x, y$ qo'shni uchlari bo'lsa, $y$ uchi, $x$ uchining "keyingi avlodi" deyiladi. $x$ uchning barcha keyingi avlodlari to'plami $S(x)$ kabi belgilanadi hamda $x \in W_n$ uchun quyidagicha aniqlanadi:

$$S(x) = \{y \in W_{n+1}: d(x, y) = 1\}.$$

Agar $x$ uchining keyingi avlodlari to'plami $S(x)$ bo'lsa, u holda barcha $x^0$dan farqli $x \in V$ uchlari uchun $|S(x)| = k$ va $|S(x^0)| = k+1$ munosabat o'rinli bo'ladi.

Biz spin qiymatlari [0,1] to'plamdagi qiymatlarni qabul qiluvchi va $\Gamma_k$ daraxtning uchlariga mos qo'yiluvchi modelni qaraymiz. Ixtiyoriy $A \subset V$ to'plam uchun $\sigma_A$ konfiguratsiya deb $\sigma_A: A \to [0,1]$ shartni qanoatlantiruvchi ixtiyoriy funksiyaga aytamiz. $A$ to'plamda aniqlangan barcha konfiguratsiyalar to'plamini $\Omega_A = [0,1]^A$ orqali belgilaymiz. Mos ravishda $x \in V \mapsto \sigma(x) \in [0,1]$ shartni qanoatlantiradigan ixtiyoriy funksiya yordamida $V$ to'plamda $\sigma$ konfiguratsiyani aniqlaymiz hamda $V$ da aniqlangan barcha konfiguratsiyalar to'plamini $[0,1]^V$ kabi belgilaymiz. Bizga [0,1] da aniqlangan uzluksiz qat'iy musbat $\varphi$ va $\psi$ funksiyalar berilgan bo'lsin. $\Gamma_k$ Keli daraxtida modelni (gamiltonian) quyidagicha aniqlaymiz:

$$H(\sigma) = -\frac{1}{\beta}\sum_{<x,y>\in L} \ln[\varphi(\sigma(x))\psi(\sigma(y)) + \varphi(\sigma(y))\psi(\sigma(x)u)]$$

bu yerda $J \in R\backslash\{0\}$ va $\xi(u, v) \in [0,1]^2 \to \xi_{uv} \in R$ berilgan, chegaralangan, o'lchovli funksiya.

Berilgan $H$ model uchun translatsion-invariant Gibbs o'lchovlari sonini topish masalasi (3) integral tenglamaning musbat yechimlarini topishga keltiriladi ([7-8] ishlarga qarang).

**Teorema.** $k \in \{2,3,4\}$ *lar uchun,* $\Gamma_k$ *Keli daraxtida aniqlangan H model yagona translatsion-invariant Gibbs o'lchoviga ega.*

**Moliyalashtirish:** *Mazkur tadqiqot ishining natijalari AL-9224093956 raqamli "Kubik stoxastik operatorlarning dinamikasi va tatbiqlari" mavzusidagi amaliy loyiha doirasida olindi.*

***ADABIYOTLAR:***

*1. Rozikov U.A. and Eshkabilov Yu.Kh. On models with uncountable set of spin values on a Cayley tree: Integral equations. Math. Phys. Anal. Geom. –2010. –13. – P.275–286.*

*2. Eshkabilov Yu.Kh., Haydarov F.H., Rozikov U.A. Non-uniqueness of Gibbs measure for models with uncountable set of spin values on a Cayley Tree. J.Stat.Phys. – 2012. – 147.– P.779– 794.*

*3. Eshkabilov Yu.Kh., Haydarov F.H., Rozikov U.A. Uniqueness of Gibbs measure for models with uncountable set of spin values on a Cayley tree. Math. Phys. Anal. Geom. – 2013.– 16(1).–P.1–17.*

*4. Eshkabilov Yu.Kh., Nodirov Sh.D., Haydarov F.H. Positive fixed points of quadratic operators and Gibbs Measures. Positivity–2016.–20(4)–P.929– 943.*

*5. Eshkabilov Yu.Kh., Nodirov Sh.D. Positive Fixed Points of Cubic Operators on R2 and Gibbs Measures. Journal of Siberian Federal University. Mathematics Physics–2019–12(6)–P.663–673.*

*6. Eshkabilov Yu.Kh., Nodirov Sh.D. On the positive fixed points of quartic operators // Bulletin of the Institute of Mathematics.–2020.–№3.– P.27– 36.*

*7. Eshkabilov Yu.Kh., Nodirov Sh.D. Positive Fixed Points of Hammerstein Integral Operators with Degenerate Kernel. Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki. 2024;166(3):437-449. (In Russ.)*

*8. Nodirov Sh.D. Description of translation-invariant Gibbs measures on a Cayley tree of arbitrary order. Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki. 2026;168(1):102-112. (In Russ.)*